



# Linearized non-minimal higher curvature supergravity

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## Abstract

In the framework of linearized non-minimal supergravity (20/20), we present the embedding of the  $R + R^2$  model and we analyze its field spectrum. As usual, the auxiliary fields of the Einstein theory now become propagating, giving rise to additional degrees of freedom, which organize themselves into on-shell irreducible supermultiplets. By performing the analysis both in component and superspace formulations we identify the new supermultiplets. On top of the two massive chiral superfields reminiscent of the old-minimal supergravity embedding, the spectrum contains also a consistent physical, massive, vector supermultiplet and a tachyonic ghost, massive, vector supermultiplet.

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## 1. Introduction

Supergravity, [1], as the low energy limit of superstring theory, offers the proper setup to study high energy gravitational phenomena. Among others, it provides an appropriate framework for the accommodation of cosmic inflation. The constraints on the latter released by the Planck collaboration [2] favor inflationary models which are characterized by plateau potentials with a

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tiny tensor-to-scalar ratio  $r$  [3]. Among the candidates is a higher curvature gravitational model, the Starobinsky model of inflation [4]

$$\sqrt{-g}^{-1} \mathcal{L} = \frac{1}{2} M_P^2 R + \frac{M_P^2}{m^2} R^2, \quad (1)$$

which stands out for its simplicity in providing a microscopic description of the mechanism responsible for the quasi de Sitter phase during inflation. This is a particular higher curvature gravitational theory of the type described in [5]. It is classically equivalent to a theory of standard gravitation coupled to an additional propagating real scalar degree of freedom [6], with a sufficiently flat potential at large values, ideal to drive inflation.

However, it is a well known fact that  $4D$ ,  $\mathcal{N} = 1$  supergravity does not have a unique off-shell description. There are two popular minimal formulations with 12 bosonic and 12 fermionic off-shell degrees of freedom (12/12): the old-minimal [7] and the new-minimal [8–10] supergravity. In addition, there exists another one with 20 bosonic and 20 fermionic off-shell degrees of freedom, which still fill an irreducible supersymmetry multiplet, the 20/20 non-minimal supergravity [11]. The Starobinsky model has been embedded in the old-minimal formulation [12–16] as well as the new-minimal formulation [15,17–20] along with various modifications [21–33]. It has also been studied in the framework of gravitino condensation [34,35]. Nevertheless there is no analogous discussion for the non-minimal formulation of supergravity. The purpose of this work is exactly that: to demonstrate the construction of the  $R + R^2$  Starobinsky model in the framework of non-minimal supergravity. For completeness, we would like to comment that there exists another non-minimal formulation [36–39] with 16/16 degrees of freedom. However it is not an irreducible representation and can be decomposed to old-minimal supergravity with a chiral supermultiplet.

To outline the procedure, we start with the free theory of nonminimal supergravity which includes a set of dynamical components that describe gravity (helicity  $\pm 2$ ) with its superpartner, the gravitino (helicity  $\pm 3/2$ ) and another set of auxiliary components just so the SUSY algebra will close off-shell. Afterwards we introduce the higher curvature terms of the form  $R^2$ . Due to the higher derivatives, the auxiliary fields of the free theory start propagating and organize themselves into supermultiplets. Nevertheless, these supermultiplets will have to be on-shell because only their dynamical degrees of freedom appear in the action, no auxiliary fields. The goal is to uncover these newly formed on-shell supermultiplets and their properties. In order to do that, we quickly realize that we do not need to start with the full theory but its linearized version will do. The results of this analysis for the case of old-minimal supergravity [12,13] are two physical, massive, chiral supermultiplets and for the case of new-minimal supergravity [17] is a physical, massive, vector supermultiplet.

Linearized supergravity is nothing else but the theory of massless, irreducible representation of the super-Poincaré group with superhelicity  $Y = 3/2$ . The superspace and component formulation of the massless, arbitrary superhelicity, irreducible representations and their properties have been studied in detail in a series of papers [40–42]. For our purpose, we will use the formalism and the results of [40] and adapt them for the case of superhelicity  $Y = 3/2$ .

The presentation of this work is organized in the following way. In Section 2 we briefly review the results of [40] for the case of linearized, non-minimal supergravity in both superspace and components. Then in Section 3 we construct the  $R^2$  action in superspace and project to components. In Section 4 we combine the two previous results to construct the Starobinsky model ( $R + R^2$ ) in this framework and study its spectrum. We perform the analysis at the component level for both bosons and fermions. At the end we verify our results by performing a duality in

superspace that reveals exactly the same spectrum and demonstrates the classical equivalence between the  $R + R^2$  theory of non-minimal supergravity and the *standard* non-minimal supergravity coupled to two massive chiral supermultiplets, a massive vector supermultiplet and a tachyonic ghost, massive vector supermultiplet. An interesting remark is that all of the massive supermultiplets turn out to have equal masses.

## 2. Superhelicity $Y = \frac{3}{2}$ as linearized non-minimal supergravity

From the investigation of free, massless, higher superspin theories [40] we can extract the  $4D$ ,  $\mathcal{N} = 1$  superspace action for linearized non-minimal supergravity

$$\begin{aligned} S_R = \int d^8z \left\{ \right. & H^{\alpha\dot{\alpha}} D^\gamma \bar{D}^2 D_\gamma H_{\alpha\dot{\alpha}} \\ & - 2H^{\alpha\dot{\alpha}} \bar{D}_{\dot{\alpha}} D^2 \chi_\alpha + c.c. \\ & - 2\chi^\alpha D^2 \chi_\alpha + c.c. \\ & \left. + 2\chi^\alpha D_\alpha \bar{D}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} \right\}, \end{aligned} \quad (2)$$

which contains the real bosonic superfield  $H_{\alpha\dot{\alpha}}$  and the fermionic superfield  $\chi_\alpha$  as a compensator. The action is invariant under the following transformation

$$\delta_G H_{\alpha\dot{\alpha}} = D_\alpha \bar{L}_{\dot{\alpha}} - \bar{D}_{\dot{\alpha}} L_\alpha, \quad (3a)$$

$$\delta_G \chi_\alpha = \bar{D}^2 L_\alpha + D^\beta \Lambda_{\alpha\beta}, \quad (3b)$$

which forces the following Bianchi identities

$$\bar{D}^{\dot{\alpha}} T_{\alpha\dot{\alpha}} - \bar{D}^2 G_\alpha = 0, \quad (4a)$$

$$\frac{1}{2!} D_{(\alpha} G_{\beta)} = 0. \quad (4b)$$

The superfields  $T_{\alpha\dot{\alpha}}$  and  $G_\alpha$  are the variations of the action (2) with respect to the unconstrained superfields  $H_{\alpha\dot{\alpha}}$  and  $\chi_\alpha$ . Their explicit expressions are

$$T_{\alpha\dot{\alpha}} = 2D^\gamma \bar{D}^2 D_\gamma H_{\alpha\dot{\alpha}} + 2 \left( D_\alpha \bar{D}^2 \bar{\chi}_{\dot{\alpha}} - \bar{D}_{\dot{\alpha}} D^2 \chi_\alpha \right), \quad (5a)$$

$$G_\alpha = -2D^2 \bar{D}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} - 4D^2 \chi_\alpha + 2D_\alpha \bar{D}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}}. \quad (5b)$$

The two superfields  $T_{\alpha\dot{\alpha}}$  and  $G_\alpha$  in (5) have mass dimensionality  $[T_{\alpha\dot{\alpha}}] = 2$ ,  $[G_\alpha] = 3/2$ .<sup>1</sup>

To prove that indeed this action describes the desired representation, using the equations of motion we can now show that a gauge invariant chiral superfield  $F_{\alpha\beta\gamma}$  exists ( $[F_{\alpha\beta\gamma}] = 5/2$ )

$$F_{\alpha\beta\gamma} = \frac{1}{3!} \bar{D}^2 D_{(\alpha} \partial_{\beta}^{\dot{\alpha}} H_{\gamma)\dot{\alpha}}, \quad (6)$$

and on-shell ( $T_{\alpha\dot{\alpha}} = G_\alpha = 0$ ), it satisfies the desired constraints in order to describe a superhelicity  $Y = 3/2$  system

$$\bar{D}_{\dot{\alpha}} F_{\alpha\beta\gamma} = 0, \quad D^\alpha F_{\alpha\beta\gamma} = 0. \quad (7)$$

<sup>1</sup> The highest spin component of  $H_{\alpha\dot{\alpha}}$  is a propagating boson.

At the component level, the above superspace action describes the dynamics of the following bosons

$$\begin{aligned}
 u_{\alpha\dot{\alpha}} &\equiv \frac{1}{2} \{D_{\alpha}\bar{G}_{\dot{\alpha}} - \bar{D}_{\dot{\alpha}}G_{\alpha}\} |, & v_{\alpha\dot{\alpha}} &\equiv -\frac{i}{2} \{D_{\alpha}\bar{G}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}}G_{\alpha}\} |, \\
 S &\equiv \frac{1}{2} \{D^{\alpha}G_{\alpha} + \bar{D}^{\dot{\alpha}}\bar{G}_{\dot{\alpha}}\} |, & P &\equiv -\frac{i}{2} \{D^{\alpha}G_{\alpha} - \bar{D}^{\dot{\alpha}}\bar{G}_{\dot{\alpha}}\} |, \\
 A_{\alpha\dot{\alpha}} &\equiv T_{\alpha\dot{\alpha}}| + \frac{1}{3} (D_{\alpha}\bar{G}_{\dot{\alpha}} - \bar{D}_{\dot{\alpha}}G_{\alpha}) |, \\
 h_{\alpha\beta\dot{\alpha}\dot{\beta}} &\equiv \frac{1}{2(2!)^2} [D_{(\alpha}, \bar{D}_{\dot{\alpha})} H_{\beta)\dot{\beta}}] |, \\
 h &\equiv \frac{1}{8} [D^{\alpha}, \bar{D}^{\dot{\alpha}}] H_{\alpha\dot{\alpha}}| + \frac{1}{2} (D^{\alpha}\chi_{\alpha} + \bar{D}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}}) |,
 \end{aligned} \tag{8}$$

namely, in 4-component notation, of three vectors  $A_{\mu}$  ( $A_{\alpha\dot{\alpha}}$ ),  $u_{\mu}$  ( $u_{\alpha\dot{\alpha}}$ ) and  $v_{\mu}$  ( $v_{\alpha\dot{\alpha}}$ ), three scalars ( $S, P, h$ ) and a symmetric traceless rank-2 tensor (the graviton)  $h_{\mu\nu}$  ( $h_{\alpha\beta\dot{\alpha}\dot{\beta}}$ ). The corresponding gauge transformations acting on the bosons are

$$\begin{aligned}
 \delta_G A_{\alpha\dot{\alpha}} &= 0, & \delta_G u_{\alpha\dot{\alpha}} &= 0, & \delta_G v_{\alpha\dot{\alpha}} &= 0, \\
 \delta_G S &= 0, & \delta_G P &= 0, \\
 \delta_G h_{\alpha\beta\dot{\alpha}\dot{\beta}} &= \frac{1}{(2!)^2} \partial_{(\alpha(\dot{\alpha}} \zeta_{\beta)\dot{\beta})}, \\
 \delta_G h &= \frac{1}{4} \partial^{\alpha\dot{\alpha}} \zeta_{\alpha\dot{\alpha}}, & \zeta_{\alpha\dot{\alpha}} &= \frac{i}{2} (D_{\alpha}\bar{L}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}}L_{\alpha}) |,
 \end{aligned} \tag{9}$$

which leave 4 degrees of freedom for each vector, 1 for each scalar and 5 for the symmetric traceless tensor, a total of 20 degrees of freedom to fill the bosonic part of the non-minimal irreducible supersymmetric multiplet. The bosonic sector of the Lagrangian density is

$$\mathcal{L}_R|_B = \mathcal{L}_{h=\pm 2} + \frac{1}{6} u^{\alpha\dot{\alpha}} u_{\alpha\dot{\alpha}} - \frac{1}{2} v^{\alpha\dot{\alpha}} v_{\alpha\dot{\alpha}} + \frac{3}{16} A^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}} - \frac{1}{8} S S - \frac{1}{8} P P, \tag{10}$$

where  $\mathcal{L}_{h=\pm 2}$  describes a massless helicity  $\pm 2$  particle

$$\begin{aligned}
 \mathcal{L}_{h=\pm 2} &= h^{\alpha\beta\dot{\alpha}\dot{\beta}} \square h_{\alpha\beta\dot{\alpha}\dot{\beta}} - h^{\alpha\beta\dot{\alpha}\dot{\beta}} \partial_{\alpha\dot{\alpha}} \partial^{\gamma\dot{\gamma}} h_{\gamma\beta\dot{\gamma}\dot{\beta}} + 2 h^{\alpha\beta\dot{\alpha}\dot{\beta}} \partial_{\alpha\dot{\alpha}} \partial_{\beta\dot{\beta}} h - 6 h \square h \\
 &= \frac{1}{\kappa^2} [\sqrt{-g} R]|_{\text{linearized}},
 \end{aligned} \tag{11}$$

and  $[\sqrt{-g} R]|_{\text{linearized}}$  is the linearized Einstein–Hilbert Lagrangian, keeping only the terms quadratic in the fields. At this linear approximation, the Ricci scalar takes the form (up to an overall normalization)

$$R = \partial^{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} h_{\alpha\beta\dot{\alpha}\dot{\beta}} - 6 \square h, \tag{12}$$

and its mass dimension is  $[R] = 3$ . The Ricci scalar is part of the completely antisymmetric  $\theta\bar{\theta}$  term in the expansion of the  $T_{\alpha\dot{\alpha}}$  superfield, specifically

$$[D^{\alpha}, \bar{D}^{\dot{\alpha}}] T_{\alpha\dot{\alpha}}| = -4R - 6\partial^{\alpha\dot{\alpha}} v_{\alpha\dot{\alpha}}. \tag{13}$$

Also, the linearized Ricci tensor is

$$R_{\alpha\beta\dot{\alpha}\dot{\beta}} = \square h_{\alpha\beta\dot{\alpha}\dot{\beta}} - \frac{1}{2!2!} \partial_{(\alpha(\dot{\alpha}} \partial^{\gamma\dot{\gamma}} h_{\gamma\beta)\dot{\gamma}\dot{\beta}} + \frac{1}{2!2!} \partial_{(\alpha(\dot{\alpha}} \partial_{\beta)\dot{\beta}} h, \tag{14}$$

and it resides in the fully symmetric part of the  $\theta\bar{\theta}$  term of  $T_{\alpha\dot{\alpha}}$

$$\frac{1}{2!2!} [D_{(\alpha}, \bar{D}_{\dot{\alpha})} T_{\beta)\dot{\beta}}] = \frac{2}{2!2!} \partial_{(\alpha(\dot{\alpha}} v_{\beta)\dot{\beta})} - 4R_{\alpha\beta\dot{\alpha}\dot{\beta}}, \quad (15)$$

while it satisfies

$$\partial^{\beta\dot{\beta}} R_{\alpha\beta\dot{\alpha}\dot{\beta}} + \frac{1}{4} \partial_{\alpha\dot{\alpha}} R = 0. \quad (16)$$

Similarly for the fermionic sector, we have the following components

$$\begin{aligned} \beta_{\alpha} &\equiv -\frac{1}{4} \left\{ D_{\alpha} \bar{D}^{\dot{\alpha}} \bar{G}_{\dot{\alpha}} - i \partial_{\alpha}^{\dot{\alpha}} \bar{G}_{\dot{\alpha}} \right\} |, \\ \rho_{\alpha} &\equiv G_{\alpha} |, \\ \psi_{\alpha\beta\dot{\alpha}} &\equiv \frac{\sqrt{2}}{2!} \bar{D}^2 D_{(\alpha} H_{\beta)\dot{\alpha}} |, \\ \psi_{\alpha} &\equiv -\sqrt{2} \left\{ D^2 \bar{D}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} + 2D^2 \chi_{\alpha} \right\} |. \end{aligned} \quad (17)$$

The gauge transformations of the fermionic fields are

$$\begin{aligned} \delta_G \rho_{\alpha} &= 0, \quad \delta_G \psi_{\alpha\beta\dot{\alpha}} = \frac{1}{2!} \partial_{(\alpha\dot{\alpha}} \xi_{\beta)}, \\ \delta_G \beta_{\alpha} &= 0, \quad \delta_G \psi_{\alpha} = -\partial_{\alpha}^{\dot{\alpha}} \bar{\xi}_{\dot{\alpha}}, \end{aligned} \quad (18)$$

with  $\xi_{\alpha} = -i\sqrt{2} \bar{D}^2 L_{\alpha} |$ . The corresponding free Lagrangian is

$$\mathcal{L}_R|_F = \mathcal{L}_{h=\pm 3/2} + \beta^{\alpha} \rho_{\alpha} + \bar{\beta}^{\dot{\alpha}} \bar{\rho}_{\dot{\alpha}}, \quad (19)$$

where  $\mathcal{L}_{h=\pm 3/2}$  describes a massless Rarita–Swinger field (gravitino with helicity  $\pm 3/2$ )

$$\mathcal{L}_{h=\pm 3/2} = i \bar{\psi}^{\alpha\dot{\alpha}\dot{\beta}} \partial^{\beta}_{\dot{\beta}} \psi_{\alpha\beta\dot{\alpha}} - \frac{3}{4} i \bar{\psi}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} \psi_{\alpha} + \left( \frac{i}{2} \psi^{\alpha\beta\dot{\alpha}} \partial_{\beta\dot{\alpha}} \psi_{\alpha} + c.c. \right). \quad (20)$$

The linearized fermionic curvatures are

$$R_{\alpha} = i\sqrt{2} \partial^{\beta\dot{\beta}} \psi_{\alpha\beta\dot{\beta}} + \frac{3i}{\sqrt{2}} \partial_{\alpha}^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}}, \quad (21a)$$

$$R_{\alpha\beta\dot{\alpha}} = \frac{i\sqrt{2}}{2!} \partial_{(\alpha}^{\dot{\beta}} \bar{\psi}_{\beta)\dot{\alpha}\dot{\beta}} + \frac{i}{\sqrt{2}2!} \partial_{(\alpha\dot{\alpha}} \psi_{\beta)}, \quad (21b)$$

and they are the (anti)symmetric part of the  $\bar{\theta}$  term of superfield  $T_{\alpha\dot{\alpha}}$

$$\frac{1}{2!} \bar{D}_{(\dot{\alpha}} T_{\alpha\dot{\beta})} = \bar{R}_{\alpha\dot{\alpha}\dot{\beta}}, \quad (22a)$$

$$\bar{D}^{\dot{\alpha}} T_{\alpha\dot{\alpha}} = R_{\alpha} - 4\beta_{\alpha} - i \partial_{\alpha}^{\dot{\alpha}} \rho_{\dot{\alpha}}. \quad (22b)$$

Finally they satisfy

$$\partial^{\alpha\dot{\alpha}} \bar{R}_{\alpha\dot{\alpha}\dot{\beta}} - \frac{1}{2} \partial^{\beta}_{\dot{\beta}} R_{\beta} = 0. \quad (23)$$

### 3. Constructing the $R^2$ theory

Now we turn to the construction of a gauge invariant, higher derivative superspace action, such that it will generate  $R^2$  terms. The reason that we restrict ourselves only to  $R^2$  terms and we do not include for example the square of the Ricci tensor, or equivalently the Weyl tensor square, is that the inclusion of the latter terms will lead to ghost and/or tachyon states in the spectrum [5,12].

To proceed in our construction, we recall that the available gauge invariant objects are the superfields  $T_{\alpha\dot{\alpha}}$ ,  $G_\alpha$  and  $F_{\alpha\beta\gamma}$ . However  $F_{\alpha\beta\gamma}$ , due to its chiral property and its index structure, it can only couple to itself, giving a term of the form  $F^{\alpha\beta\gamma} F_{\alpha\beta\gamma}$ . But such an object will give rise to the square of the Weyl tensor, so it is rejected. The rest of the objects could be combined in many different ways. We organize them in the following manner.

The general structure of all possible terms that we are interested in, are schematically of the form

$$T^n D^k G^l, \quad (24)$$

which means that any possible term will include  $n$   $T_{\alpha\dot{\alpha}}$ 's,  $k$  superspace covariant derivatives and  $l$   $G_\alpha$ 's. The dimensionality of these terms is

$$2n + \frac{k}{2} + \frac{3l}{2}. \quad (25)$$

Then, if we project to components, we have to integrate over superspace  $\bar{D}^2 D^2 (T^n D^k G^l)$ , and therefore the mass dimension of the component terms that we can, in principle, construct is

$$2n + \frac{k}{2} + \frac{3l}{2} + 2. \quad (26)$$

The final step is the fact that the desired  $R^2$  term has dimensionality 6 and we require to have expressions quadratic in the components (linear approximation). Therefore we must have

$$2n + \frac{k}{2} + \frac{3l}{2} + 2 = 6, \quad (27a)$$

$$n + l = 2. \quad (27b)$$

The solutions of this Diophantine system, and the corresponding terms allowed are given in the following table

| $n$ | $k$ | $l$ | term  |
|-----|-----|-----|---|
| 2   | 0   | 0   | $T^{\alpha\dot{\alpha}} T_{\alpha\dot{\alpha}}$                   |
| 1   | 1   | 1   | $T^{\alpha\dot{\alpha}} \bar{D}_{\dot{\alpha}} G_\alpha + c.c.$   |
| 0   | 2   | 2   | $G^\alpha D_\alpha \bar{D}^{\dot{\alpha}} \bar{G}_{\dot{\alpha}}$ |
|     |     |     | $G^\alpha \bar{D}^{\dot{\alpha}} D_\alpha \bar{G}_{\dot{\alpha}}$ |
|     |     |     | $G^\alpha \bar{D}^2 G_\alpha + c.c.$                              |

Note that we have not included the term  $G^\alpha \bar{D}^2 G_\alpha + c.c.$  since it is zero due to (4b). Moreover because of Eq. (4a) the terms  $T^{\alpha\dot{\alpha}} \bar{D}_{\dot{\alpha}} G_\alpha + c.c.$  and  $G^\alpha \bar{D}^2 G_\alpha + c.c.$  are identical.

Hence the  $R^2$  superspace action must be of the form

$$S_{R^2} = \int d^8z \left\{ g_0 T^{\alpha\dot{\alpha}} T_{\alpha\dot{\alpha}} + g_1 G^\alpha D_\alpha \bar{D}^{\dot{\alpha}} \bar{G}_{\dot{\alpha}} + g_2 G^\alpha \bar{D}^{\dot{\alpha}} D_\alpha \bar{G}_{\dot{\alpha}} + (g_3 G^\alpha \bar{D}^2 G_\alpha + c.c.) \right\} \quad (28)$$

where  $g_0, g_1, g_2 \in \mathbb{R}$ . Now what remains is to project this action to components and pick the coefficients in a way such that we generate  $R^2$  terms and canonical kinematic terms for any additional propagating fields.

The component Lagrangian we get from the above action (28) is

$$\begin{aligned} \mathcal{L}_{R^2} = & g_0 \bar{D}^2 D^2 (T^{\alpha\dot{\alpha}} T_{\alpha\dot{\alpha}}) + g_1 \bar{D}^2 D^2 (G^\alpha D_\alpha \bar{D}^{\dot{\alpha}} \bar{G}_{\dot{\alpha}}) \\ & + g_2 \bar{D}^2 D^2 (G^\alpha \bar{D}^{\dot{\alpha}} D_\alpha \bar{G}_{\dot{\alpha}}) + \left\{ g_3 \bar{D}^2 D^2 (G^\alpha \bar{D}^2 G_\alpha) + c.c. \right\}. \end{aligned} \quad (29)$$

The basic rules for projection are

1. Use the ‘Leibniz’ rule

$$\begin{aligned} \bar{D}^2 D^2 (AB) = & \bar{D}^2 D^2 A|B| + (-1)^{\epsilon(A)} \bar{D}^\rho D^2 A|\bar{D}_\rho B| + D^2 A|\bar{D}^2 B| \\ & + (-1)^{\epsilon(A)} \bar{D}^2 D^\rho A|D_\rho B| - \bar{D}^\rho D^\rho A|\bar{D}_\rho D_\rho B| \\ & + (-1)^{\epsilon(A)} D^\rho A|\bar{D}^2 D_\rho B| \\ & + \bar{D}^2 A|D^2 B| + (-1)^{\epsilon(A)} \bar{D}^\rho A|\bar{D}_\rho D^2 B| + A|\bar{D}^2 D^2 B|, \end{aligned} \quad (30)$$

where  $\epsilon$  is zero for bosonic and one for fermionic superfields.

2. Use the Bianchi identities (4).
3. Use the component definitions of (8) and (17).

First we focus on the bosonic sector of the theory, therefore we restrict the above calculation to the bosonic part of the projection. That means, we keep only the terms with even number of  $D$ ’s when acting on a bosonic superfield (like  $T_{\alpha\dot{\alpha}}$ ) and with odd number of  $D$ ’s when acting on a fermionic superfield (like  $G_\alpha$ ). We get

$$\mathcal{L}_{R^2}|_B = I_0|_B + I_1|_B + I_2|_B + I_3|_B, \quad (31)$$

with

$$I_0|_B = g_0 \bar{D}^2 D^2 (T^{\alpha\dot{\alpha}} T_{\alpha\dot{\alpha}})|_B, \quad (32a)$$

$$I_1|_B = g_1 \bar{D}^2 D^2 (G^\alpha D_\alpha \bar{D}^{\dot{\alpha}} \bar{G}_{\dot{\alpha}})|_B, \quad (32b)$$

$$I_2|_B = g_2 \bar{D}^2 D^2 (G^\alpha \bar{D}^{\dot{\alpha}} D_\alpha \bar{G}_{\dot{\alpha}})|_B, \quad (32c)$$

$$I_3|_B = g_3 \bar{D}^2 D^2 (G^\alpha \bar{D}^2 G_\alpha)|_B + c.c. \quad (32d)$$

It is evident that  $I_0|_B$  includes a term proportional to  $[D^{(\rho}, \bar{D}^{(\dot{\rho}}] T^{\alpha\dot{\alpha}}) [D_{(\rho}, \bar{D}_{(\dot{\rho}}] T_{\alpha\dot{\alpha}})$ , which based on (15) makes it obvious that it generates the Ricci tensor square,  $R^{\alpha\beta\dot{\alpha}\dot{\beta}} R_{\alpha\beta\dot{\alpha}\dot{\beta}}$ , a term that is not considered here (as it leads to ghost and/or tachyonic states [5,12]). On top of that, such a term cannot be canceled by any of the other contributions to the bosonic Lagrangian. Therefore the only possibility out of that is to choose

$$g_0 = 0. \quad (33)$$

The rest of the terms are relevant and after putting everything together, we find that the total bosonic sector is

$$\begin{aligned}
\mathcal{L}_{R^2}|_B = & \frac{1}{4} \left[ g_1 - g_2 - g_3^R \right] A^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} A_{\beta\dot{\beta}} \\
& + \frac{1}{6} \left[ 4g_1 - g_2 + 2g_3^R \right] A^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} u_{\beta\dot{\beta}} \\
& + \frac{1}{9} \left[ 4g_1 - 7g_2 + 8g_3^R \right] u^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} u_{\beta\dot{\beta}} + \left[ g_2 - 2g_3^R \right] u^{\alpha\dot{\alpha}} \square u_{\alpha\dot{\alpha}} \\
& + \left[ 4g_1 - 7g_2 + 8g_3^R \right] v^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} v_{\beta\dot{\beta}} + \left[ g_2 + 2g_3^R \right] v^{\alpha\dot{\alpha}} \square v_{\alpha\dot{\alpha}} \\
& + 2 \left[ 4g_1 - 5g_2 + 6g_3^R \right] v^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} R + \left[ 3g_3^I \right] A^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} v_{\beta\dot{\beta}} \\
& + \left[ -g_1 + \frac{1}{2}g_2 \right] S \square S + \left[ 2g_3^I \right] A^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} R \\
& + \left[ -g_1 + \frac{1}{2}g_2 \right] P \square P + \left[ -4g_3^I \right] u^{\alpha\dot{\alpha}} \square v_{\alpha\dot{\alpha}} \\
& - 4 \left[ g_1 - g_2 + g_3^R \right] R^2 + \left[ -\frac{4}{3}g_3^I \right] u^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} R, \tag{34}
\end{aligned}$$

where  $g_3^R$  and  $g_3^I$  are the real and imaginary parts of  $g_3$ . Notice that the higher curvature terms are accompanied by kinematic terms for all the previously auxiliary fields. This is a standard property of higher curvature supergravity.

Similarly, we find that the fermionic sector is

$$\begin{aligned}
\mathcal{L}_{R^2}|_F = & -4 \left[ 4g_1 - 7g_2 + 8g_3^R \right] i \bar{\beta}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} \beta_{\alpha} \\
& - \frac{1}{4} \left[ 4g_1 - 7g_2 + 8g_3^R \right] i \bar{\rho}^{\dot{\alpha}} \square \partial^{\alpha}_{\dot{\alpha}} \rho_{\alpha} \\
& + \left[ 4g_1 + 3g_2 - 8g_3^R \right] \beta^{\alpha} \square \rho_{\alpha} + c.c. \\
& + \left[ g_2 \right] i \bar{R}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} R_{\alpha} \\
& - 4 \left[ g_2 - g_3 \right] i \bar{\beta}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} R_{\alpha} + c.c. \\
& - \left[ g_2 - g_3 \right] \rho^{\alpha} \square R_{\alpha} + c.c. \tag{35}
\end{aligned}$$

#### 4. The spectrum of $R + R^2$ non-minimal supergravity

So far we have developed the superspace action for the  $R$  and  $R^2$  theories. In this section we combine them in order to study the spectrum of the  $R + R^2$  theory. Specifically we will analyze the propagating degrees of freedom of the Lagrangian

$$\mathcal{L} = \mathcal{L}_R + \frac{1}{m^2} \mathcal{L}_{R^2}. \tag{36}$$

To do this we must first bring the full Lagrangian into a diagonal form and subsequently study their field equations. Typically one can achieve that by doing redefinitions of the various fields and a clever choice of coefficients. But, in this case due to the fact that the  $\mathcal{L}_R$  is already diagonal, we cannot perform any redefinitions and the only thing left to do is to choose appropriately the coefficients of the non-diagonal terms.



#### 4.1. Bosonic sector spectrum

Following the previously explained strategy, we must impose the constraints

$$\left. \begin{aligned} 4g_1 - g_2 + 2g_3^R &= 0 \\ 4g_1 - 5g_2 + 6g_3^R &= 0 \\ g_3^I &= 0 \end{aligned} \right\} \quad g_1 = -\frac{1}{4}g, \quad g_2 = g_3 = g, \quad g \in \mathbb{R}. \quad (37)$$

With the above coefficients (37), we find that the linearized, bosonic part of the component Lagrangian is

$$\begin{aligned} \mathcal{L}|_B &= \mathcal{L}_{h=\pm 2} + \frac{g}{m^2} R^2 \\ &+ \frac{3}{16} A^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}} - \frac{9}{16} \frac{g}{m^2} A^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} A_{\beta\dot{\beta}} \\ &+ \frac{1}{6} u^{\alpha\dot{\alpha}} u_{\alpha\dot{\alpha}} - \frac{g}{m^2} u^{\alpha\dot{\alpha}} \square u_{\alpha\dot{\alpha}} \\ &- \frac{1}{2} v^{\alpha\dot{\alpha}} v_{\alpha\dot{\alpha}} + 3 \frac{g}{m^2} v^{\alpha\dot{\alpha}} \square v_{\alpha\dot{\alpha}} \\ &- \frac{1}{8} S^2 + \frac{3}{4} \frac{g}{m^2} S \square S \\ &- \frac{1}{8} P^2 + \frac{3}{4} \frac{g}{m^2} P \square P. \end{aligned} \quad (38)$$

The equations of motion for the various fields and the degrees of freedom they allow to propagate are:

1. For  $A_{\alpha\dot{\alpha}}$  we have

$$A_{\alpha\dot{\alpha}} - 3 \frac{g}{m^2} \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} A_{\beta\dot{\beta}} = 0 \rightsquigarrow \square \partial^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}} = \frac{m^2}{6g} \partial^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}}. \quad (39)$$

From the left equation we see that three of the degrees of freedom of the vector field  $A_{\alpha\dot{\alpha}}$  remain auxiliary and are solved in terms of the scalar  $\partial^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}}$ . From the right equation we see that for  $g > 0$ ,  $\partial^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}}$  is a physical, real, propagating, massive scalar with mass  $\mu^2 = m^2/6g$ .

2. For  $u_{\alpha\dot{\alpha}}$  we find

$$\frac{1}{6} u_{\alpha\dot{\alpha}} - \frac{g}{m^2} \square u_{\alpha\dot{\alpha}} = 0 \rightsquigarrow \square u_{\alpha\dot{\alpha}} = \frac{m^2}{6g} u_{\alpha\dot{\alpha}}. \quad (40)$$

This describes the propagation of a real, massive, scalar  $\partial^{\alpha\dot{\alpha}} u_{\alpha\dot{\alpha}}$  with equations of motion  $\square \partial^{\alpha\dot{\alpha}} u_{\alpha\dot{\alpha}} = \frac{m^2}{6g} \partial^{\alpha\dot{\alpha}} u_{\alpha\dot{\alpha}}$  and mass  $\mu^2 = m^2/6g$ , and the propagation of a real, massive vector with the same mass described by the divergent-less field defined as  $\hat{u}_{\alpha\dot{\alpha}} = u_{\alpha\dot{\alpha}} - \frac{3g}{m^2} \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} u_{\beta\dot{\beta}}$ , with equations of motion  $\square \hat{u}_{\alpha\dot{\alpha}} = \frac{m^2}{6g} \hat{u}_{\alpha\dot{\alpha}}$ . Both of them are tachyonic ghosts (for  $g > 0$ ) since they appear in the Lagrangian with an opposite overall sign.

3. For  $v_{\alpha\dot{\alpha}}$  we have

$$-\frac{1}{2} v_{\alpha\dot{\alpha}} + 3 \frac{g}{m^2} \square v_{\alpha\dot{\alpha}} = 0 \rightsquigarrow \square v_{\alpha\dot{\alpha}} = \frac{m^2}{6g} v_{\alpha\dot{\alpha}}. \quad (41)$$

As before this equation includes both the spin zero part, described by  $\partial^{\alpha\dot{\alpha}}v_{\alpha\dot{\alpha}}$  and the spin one part, described by the  $\hat{v}_{\alpha\dot{\alpha}} = v_{\alpha\dot{\alpha}} - \frac{3g}{m^2}\partial_{\alpha\dot{\alpha}}\partial^{\beta\dot{\beta}}v_{\beta\dot{\beta}}$ . Both of them have the same mass  $\mu^2 = m^2/6g$  and are physical for  $g > 0$ .

4. For  $S$  we find

$$-\frac{1}{2}S + 3\frac{g}{m^2}\square S \rightsquigarrow \square S = \frac{m^2}{6g}S, \quad (42)$$

which describes a physical ( $g > 0$ ), real, massive propagating scalar with mass  $\mu^2 = m^2/6g$ .

5. For  $P$  we find

$$-\frac{1}{2}P + 3\frac{g}{m^2}\square P \rightsquigarrow \square P = \frac{m^2}{6g}P. \quad (43)$$

Same as  $S$ , it describes a physical ( $g > 0$ ), real, massive propagating scalar with mass  $\mu^2 = m^2/6g$ .

6. The gravitational sector of the action is

$$\mathcal{S}'|_B = \int d^4x \mathcal{L}_{h=\pm 2} + \frac{g}{m^2} \int d^4x R^2, \quad (44)$$

which can be re-expressed with the help of a Lagrange multiplier  $\phi$  in the following form

$$\mathcal{S}'|_B = \int d^4x \mathcal{L}_{h=\pm 2} + f \int d^4x \phi R - \frac{f^2}{4g} m^2 \int d^4x \phi^2, \quad (45)$$

where  $[\phi] = 1$ . Now we perform the following redefinition of  $h$

$$h \rightarrow h + c\phi. \quad (46)$$

The change of  $\mathcal{L}_{h=\pm 2}$  is

$$\delta \mathcal{L}_{h=\pm 2} = 2c\phi R - 6c^2\phi\square\phi, \quad (47)$$

and the change of  $R$  is

$$\delta R = -6c\square\phi. \quad (48)$$

Therefore we get for  $\mathcal{S}'|_B$

$$\begin{aligned} \mathcal{S}'|_B = & \int d^4x \mathcal{L}_{h=\pm 2} + (2c + f) \int d^4x \phi R \\ & - 6c(c + f) \int d^4x \phi\square\phi - \frac{f^2}{4g} m^2 \int d^4x \phi^2. \end{aligned} \quad (49)$$

We choose  $c$  such that the cross term vanish

$$2c + f = 0, \quad (50)$$

hence we get

$$\begin{aligned} \mathcal{S}'|_B = & \int d^4x \mathcal{L}_{h=\pm 2} \\ & + \frac{3}{2}f^2 \int d^4x \phi\square\phi - \frac{f^2}{4g} m^2 \int d^4x \phi^2, \end{aligned} \quad (51)$$

which describes a helicity  $\pm 2$  and a physical (for  $g > 0$ ), real, massive, scalar  $\phi$  with mass  $\mu^2 = m^2/(6g)$ .

To summarize, beside the helicity  $\pm 2$  system, the spectrum organizes into two physical massive chiral supermultiplets  $(\partial^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}}, \phi)$  and  $(S, P)$ , one physical massive vector supermultiplet  $(\hat{v}_{\alpha\dot{\alpha}}, \partial^{\alpha\dot{\alpha}} v_{\alpha\dot{\alpha}})$  and one tachyonic-ghost massive vector supermultiplet  $(\hat{u}_{\alpha\dot{\alpha}}, \partial^{\alpha\dot{\alpha}} u_{\alpha\dot{\alpha}})$ .

#### 4.2. Fermionic sector spectrum

In order to verify the fermionic spectrum, we start with Eq. (35) and make the same choice of coefficients as in (37), which give

$$\begin{aligned} \mathcal{L}|_F = & \mathcal{L}_{h=\pm 3/2} + \beta^\alpha \rho_\alpha - 6 \frac{g}{m^2} \beta^\alpha \square \rho_\alpha \\ & + \bar{\beta}^{\dot{\alpha}} \bar{\rho}_{\dot{\alpha}} - 6 \frac{g}{m^2} \bar{\beta}^{\dot{\alpha}} \square \bar{\rho}_{\dot{\alpha}} + i \frac{g}{m^2} \bar{R}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} R_\alpha. \end{aligned} \quad (52)$$

The equations of motion for the various fields are

1. From  $\beta_\alpha$  and  $\bar{\beta}_{\dot{\alpha}}$  we find

$$\square \rho_\alpha = \frac{m^2}{6g} \rho_\alpha, \quad \square \bar{\rho}_{\dot{\alpha}} = \frac{m^2}{6g} \bar{\rho}_{\dot{\alpha}}, \quad (53)$$

which describe a pair of massive Weyl spinors with Dirac mass  $\mu^2 = m^2/(6g)$ .

2. From  $\rho_\alpha$  and  $\bar{\rho}_{\dot{\alpha}}$  we find

$$\square \beta_\alpha = \frac{m^2}{6g} \beta_\alpha, \quad \square \bar{\beta}_{\dot{\alpha}} = \frac{m^2}{6g} \bar{\beta}_{\dot{\alpha}}, \quad (54)$$

which again describe a pair of massive Weyl spinors with Dirac mass  $\mu^2 = m^2/(6g)$ . Note that, in order to reveal the fermions that belong to the tachyonic-ghost vector multiplet, we have to diagonalize the Lagrangian (52). Once we do that, we will get one positive and one negative eigenvalue, which signals the propagation of one physical and one tachyonic-ghost fermion.

3. The rest of the action includes  $\mathcal{L}_{h=\pm 3/2}$  and  $R_\alpha$  and can be expressed in the following way

$$\begin{aligned} S'|_F = & \int d^4x \mathcal{L}_{h=\pm 3/2} + i g \int d^4x \bar{\zeta}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} \zeta_\alpha \\ & + m \int d^4x \phi^\alpha \left\{ \zeta_\alpha - \frac{R_\alpha}{m} \right\} + c.c. \end{aligned} \quad (55)$$

Now we redefine  $\psi_\alpha$

$$\psi_\alpha \rightarrow \psi_\alpha + d\phi_\alpha. \quad (56)$$

The change of  $\mathcal{L}_{h=\pm 3/2}$  is

$$\begin{aligned} \delta \mathcal{L}_{h=\pm 3/2} = & -\frac{d}{2\sqrt{2}} \phi^\alpha R_\alpha + c.c. \\ & - \frac{3}{4} |d|^2 i \bar{\phi}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} \phi_\alpha, \end{aligned} \quad (57)$$

and the change of  $R_\alpha$  is

$$\delta R_\alpha = \frac{3\bar{d}}{\sqrt{2}} i \partial^{\dot{\alpha}}_{\alpha} \bar{\phi}_{\dot{\alpha}}. \quad (58)$$

So we get that

$$\begin{aligned} \mathcal{S}'|_F = & \int d^4x \mathcal{L}_{h=\pm 3/2} + ig \int d^4x \bar{\zeta}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} \zeta_{\alpha} + m \int d^4x \left\{ \phi^{\alpha} \zeta_{\alpha} + \bar{\phi}^{\dot{\alpha}} \bar{\zeta}_{\dot{\alpha}} \right\} \\ & - \left( \frac{d}{2\sqrt{2}} + 1 \right) \int d^4x \phi^{\alpha} R_{\alpha} + c.c. \\ & - \left( \frac{3}{4} |l|^2 + \frac{6\bar{d}}{\sqrt{2}} \right) i \int d^4x \bar{\phi}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} \phi_{\alpha}. \end{aligned} \quad (59)$$

Finally we choose  $d$  in order to cancel the interaction term with  $R_{\alpha}$

$$d + 2\sqrt{2} = 0, \quad (60)$$

and we get

$$\begin{aligned} \mathcal{S}'|_F = & \int d^4x \mathcal{L}_{h=\pm 3/2} + ig \int d^4x \bar{\zeta}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} \zeta_{\alpha} + 6i \int d^4x \bar{\phi}^{\dot{\alpha}} \partial^{\alpha}_{\dot{\alpha}} \phi_{\alpha} \\ & + m \int d^4x \left\{ \phi^{\alpha} \zeta_{\alpha} + \bar{\phi}^{\dot{\alpha}} \bar{\zeta}_{\dot{\alpha}} \right\}. \end{aligned} \quad (61)$$

The equations of motion from Lagrangian (61) on top of the massless gravitino, give two massive Weyl spinors with Dirac mass  $\mu^2 = m^2/(6g)$ .

Therefore the spectrum of fermions gives, as expected, the same structure.

### 4.3. Superspace duality

From our previous considerations, we find that this higher curvature theory has additional propagating degrees of freedom. Since this is a supersymmetric theory it should be possible to identify the multiplet structure of these new degrees of freedom directly from superspace manipulations. In other words we expect to find that our higher curvature theory is classically equivalent to a particular set of matter fields coupled to *standard* non-minimal supergravity (i.e. a supergravity with no higher curvature terms). The Superspace action for the above choice of coefficients is of the form

$$\begin{aligned} \mathcal{S} = & \mathcal{S}_R - \frac{1}{4} \frac{g}{m^2} \int d^8z G^{\alpha} D_{\alpha} \bar{D}^{\dot{\alpha}} \bar{G}_{\dot{\alpha}} \\ & + \frac{g}{m^2} \int d^8z G^{\alpha} \bar{D}^{\dot{\alpha}} D^{\alpha} \bar{G}_{\dot{\alpha}} \\ & + \frac{g}{m^2} \int d^8z G^{\alpha} \bar{D}^2 \bar{G}_{\dot{\alpha}} + c.c. \\ = & \mathcal{S}_R + \frac{g}{4m^2} \int d^8z \bar{\Phi} \Phi - \frac{g}{2m^2} \int d^8z V^{\alpha\dot{\alpha}} V_{\alpha\dot{\alpha}}, \end{aligned} \quad (62)$$

for the chiral  $\Phi = \bar{D}^{\dot{\alpha}} \bar{G}_{\dot{\alpha}}$  and the real vector  $V_{\alpha\dot{\alpha}} = i(D_{\alpha} \bar{G}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}} G_{\alpha})$ . The action (62) can be re-written as

$$\begin{aligned} \mathcal{S} = & \mathcal{S}_R + mk \int d^8z T \left( S - \frac{\Phi}{m} \right) + mk \int d^8z \bar{T} \left( \bar{S} - \frac{\bar{\Phi}}{m} \right) + \frac{g}{4} \int d^8z \bar{S} S \\ & + l \int d^8z F^{\alpha\dot{\alpha}} V_{\alpha\dot{\alpha}} + m^2 \frac{l^2}{2g} \int d^8z F^{\alpha\dot{\alpha}} F_{\alpha\dot{\alpha}}, \end{aligned} \quad (63)$$

where  $T$  ( $[T] = 0$ ) is an unconstrained scalar superfield,  $S$  ( $[S] = 1$ ) is a chiral superfield and  $F_{\alpha\dot{\alpha}}$  ( $[F_{\alpha\dot{\alpha}}] = 0$ ) is a real vector superfield. Indeed, the equations of motion of  $T$  and  $F_{\alpha\dot{\alpha}}$  lead to the original action (62). Now we perform the following shift

$$\chi_\alpha \rightarrow \chi_\alpha + c D_\alpha T + i d \bar{D}^{\dot{\alpha}} F_{\alpha\dot{\alpha}}, \quad (64)$$

under which we find

$$\begin{aligned} \mathcal{S} = & \mathcal{S}_R + mk \int d^8 z \{ T S + \bar{T} \bar{S} \} + \frac{g}{m^2} \int d^8 z \bar{S} S + m^2 \frac{l^2}{2g} \int d^8 z F^{\alpha\dot{\alpha}} F_{\alpha\dot{\alpha}} \\ & - [k + c] \int d^8 z \{ T \Phi + \bar{T} \bar{\Phi} \} + [l + d] \int d^8 z F^{\alpha\dot{\alpha}} V_{\alpha\dot{\alpha}} \\ & + [4kd + 4cd + 4lc] \int d^8 z T \bar{D}^2 \partial^{\alpha\dot{\alpha}} F_{\alpha\dot{\alpha}} + c.c. \\ & - [16kc + 8c^2] \int d^8 z T \bar{D}^2 D^2 \bar{T} \\ & + \left[ \frac{d^2}{2} + ld \right] \int d^8 z \left\{ F^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}] [D^\beta, \bar{D}^{\dot{\beta}}] F_{\beta\dot{\beta}} + 3 \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} F_{\beta\dot{\beta}} \right\}. \end{aligned} \quad (65)$$

We now choose coefficients  $c$  and  $d$  to eliminate the cross terms involving superfields  $\Phi$  and  $V_{\alpha\dot{\alpha}}$  respectively, which gives  $c = -k$  and  $d = -l$ , leading to

$$\begin{aligned} \mathcal{S} = & \mathcal{S}_R + mk \int d^8 z \{ T S + \bar{T} \bar{S} \} + \frac{g}{m^2} \int d^8 z \bar{S} S + m^2 \frac{l^2}{2g} \int d^8 z F^{\alpha\dot{\alpha}} F_{\alpha\dot{\alpha}} \\ & - 4lk \left( \int d^8 z T \bar{D}^2 \partial^{\alpha\dot{\alpha}} F_{\alpha\dot{\alpha}} + c.c. \right) + 8k^2 \int d^8 z T \bar{D}^2 D^2 \bar{T} \\ & - \frac{l^2}{2} \int d^8 z \left\{ F^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}] [D^\beta, \bar{D}^{\dot{\beta}}] F_{\beta\dot{\beta}} + 3 \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} F_{\beta\dot{\beta}} \right\}. \end{aligned} \quad (66)$$

It is obvious that the above action contains linearized non-minimal supergravity with no higher curvature terms and an independent additional matter sector. Before we conclude let us study the on-shell superfield content of the matter sector, and compare to our findings from the component discussion.

The equations of motion for superfields  $F_{\alpha\dot{\alpha}}$ ,  $T$ ,  $S$  are

$$\begin{aligned} \mathcal{E}_{\alpha\dot{\alpha}}^{(F)} = & -l^2 [D_\alpha, \bar{D}_{\dot{\alpha}}] [D^\beta, \bar{D}^{\dot{\beta}}] F_{\beta\dot{\beta}} - 3l^2 \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} F_{\beta\dot{\beta}} \\ & + 4lk \partial_{\alpha\dot{\alpha}} \left( \bar{D}^2 T + D^2 \bar{T} \right) + \frac{l^2}{g} m^2 F_{\alpha\dot{\alpha}}, \end{aligned} \quad (67a)$$

$$\mathcal{E}^T = 8k^2 \bar{D}^2 D^2 \bar{T} - 4lk \bar{D}^2 \partial^{\alpha\dot{\alpha}} F_{\alpha\dot{\alpha}} + mk S, \quad (67b)$$

$$\mathcal{E}^S = -\frac{g}{4} \bar{D}^2 \bar{S} - mk \bar{D}^2 T. \quad (67c)$$

Looking for the solution of the above equations, we do the following ansatz:

$$F_{\alpha\dot{\alpha}} = \partial_{\alpha\dot{\alpha}} V + [D_\alpha, \bar{D}_{\dot{\alpha}}] W + \frac{1}{m^2} \partial_{\alpha\dot{\alpha}} \left( \bar{D}^2 T + D^2 \bar{T} \right), \quad (68)$$

where  $V$  and  $W$  are on-shell, real, superfields which they satisfy the constraints  $D^2 V = \bar{D}^2 V = 0$ ,  $D^2 W = \bar{D}^2 W = 0$  and we have for their equations of motion

$$D^\gamma \bar{D}^2 D_\gamma V + \kappa_V m^2 V = 0, \quad D^\gamma \bar{D}^2 D_\gamma W + \kappa_W m^2 W = 0. \quad (69)$$

By doing that, we realize that there are two on-shell chiral supermultiplets, described by the chiral superfields  $\bar{D}^2 T$  and  $S$  and they satisfy the following equations of motion

$$\square(\bar{D}^2 T) = \kappa_T m^2 (\bar{D}^2 T), \quad \square S = \kappa_S m^2 S. \quad (70)$$

The above equations (69) and (70) solve the system (67) if we set

$$\kappa_V = \kappa_W = \kappa_T = \kappa_S = \frac{1}{6g}, \quad k = -\frac{l}{12g}. \quad (71)$$

From (69) and (70) we see that indeed we get two vector supermultiplets and two chiral supermultiplets with equal masses  $\mu^2 = \frac{m^2}{6g}$ . The final expression for the superspace action is

$$\begin{aligned} S = S_R - \frac{l^2}{12} \frac{m}{g} \int d^8 z \{ T S + \bar{T} \bar{S} \} + \frac{g}{m^2} \int d^8 z \bar{S} S + m^2 \frac{l^2}{2g} \int d^8 z F^{\alpha\dot{\alpha}} F_{\alpha\dot{\alpha}} \\ + \frac{l^2}{18g^2} \int d^8 z T \bar{D}^2 D^2 \bar{T} + \left( \frac{l^2}{3g} \int d^8 z T \bar{D}^2 \partial^{\alpha\dot{\alpha}} F_{\alpha\dot{\alpha}} + c.c. \right) \\ - \frac{l^2}{2} \int d^8 z \left\{ F^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}] [D^\beta, \bar{D}^{\dot{\beta}}] F_{\beta\dot{\beta}} + 3 \partial_{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} F_{\beta\dot{\beta}} \right\} \end{aligned} \quad (72)$$

where  $g$  and  $l$  are free, non-zero parameters. Furthermore, due to the different integration by parts properties of the two operators  $\partial_{\alpha\dot{\alpha}}$  and  $[D_\alpha, \bar{D}_{\dot{\alpha}}]$ , we immediately conclude that there will be an overall minus in front of the terms quadratic to  $W$ , illustrating that the  $W$  massive vector supermultiplet will be a tachyonic ghost one. The above performed superspace duality demonstrated the classical equivalence between the higher curvature non-minimal supergravity theory and the non-minimal supergravity with the addition of a specific spectrum that we are expecting from the previous component discussions.

## 5. Conclusions

In this work we have studied the spectrum of the Starobinsky model  $R + R^2$ , embedded in the framework of non-minimal supergravity. We have utilized the linearized theory since it is sufficient for the understanding of the field content. As expected from a supergravity theory, on top of the scalaron degree of freedom, there are previously auxiliary fields which now pick up kinematic terms due to the new action. We have identified these fields and the way they organize inside supermultiplets. Our findings show that the 20/20 higher curvature supergravity is classically equivalent to a 20/20 supergravity coupled to two vector supermultiplets (one of which is a tachyonic ghost multiplet) and two chiral supermultiplets with equal masses. Therefore, the embedding of the  $R + R^2$  theory in non-minimal supergravity is reminiscent of the corresponding embedding of the general quadratic gravity (with  $R^2$  and Weyl square terms) in minimal supergravity, as in both cases unphysical states appear in the spectrum.

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